ANALYSIS OF THE SUBSONIC SEPARATION GAS FLOW AROUND A WING–FUSELAGE SYSTEM

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Problems of subsonic separation gas flow around a small span wing as well as a finite span wing with an extension were considered in [1, 2]. It is shown on the basis of an asymptotic analysis of these problems that the Goethert rule can be used in both cases even in the following approximation (nonlinear in the angle of attack). In this paper these results are extended to the case of separation flow around a wing–fuselage combination and the fundamental statements of an asymptotic analysis of the problems under investigation are elucidated briefly. A detailed description of the mathematical apparatus is given in [1, 2]. An alternative approach based on the direct solution of the Euler equation by an iteration method without using information about the smallness of the parameters is developed in [3].

1. Formulation of the Problem

Let us consider stationary separation gas flow around the combination of a zero-thickness wing and a fuselage for $0 \le M_\infty < 1$. Let us select the system of measurement units in such a way that the free-stream velocity and the central chord of the wing equal one. It is assumed that the wing and fuselage surfaces are symmetric relative to the xy plane and there is no slip. We assume that there are vortical surfaces (vortex sheet) in the flow that leave the wing leading edges but there is no separation from the fuselage.

We make the following assumptions relative to the geometry of the components. Let the fuselage length be O(1) and its transverse dimension $\varepsilon = o(1)$. Let us examine separately the cases of a small-span wing $\lambda = O(\varepsilon)$ (problem 1) and a finite-span wing $\lambda = O(1)$ with a high sweepback extension (problem 2) so that the span of the extension $\lambda_e = O(\varepsilon)$. In both cases it is assumed that $\alpha = O(\varepsilon)$.

Let φ be the flow potential, a the speed of sound, and u, v, w the velocity components along the x, y, z, axes. Then the flow potential satisfies the equation [4]

$$\left(1-\frac{u^2}{a^2}\right)\frac{\partial^2\varphi}{\partial x^2}+\left(1-\frac{v^2}{a^2}\right)\frac{\partial^2\varphi}{\partial y^2}+\left(1-\frac{w^2}{a^2}\right)\frac{\partial^2\varphi}{\partial z^2}-2\frac{uv}{a^2}\frac{\partial^2\varphi}{\partial x\partial y}-2\frac{uw}{a^2}\frac{\partial^2\varphi}{\partial x\partial z}-2\frac{vw}{a^2}\frac{\partial\varphi^2}{\partial y\partial z}=0,$$

the boundary conditions of nonpenetration on the fuselage, wing, and sheet, as well as the continuity for the pressure during passage through the sheet and the Chaplygin–Zhukovskii condition on those edges where the presence of flow separation is assumed.

2. Asymptotic Analysis

We turn to a description of the results of an asymptotic analysis of problem 1 (the fuselage–small wingspan combination). A complete elucidation of the mathematical apparatus is presented in [1].

Let us introduce zone 1, the exterior flow domain with characteristic dimensions O(1) in which for $\lambda \to 0$ and $\varepsilon = O(\lambda)$ the wing–fuselage system is mapped by a segment of the x axis. Then in a first approximation we have a simply uniform flow in zone 1. This solution does not describe the flow in the neighborhood of the wing, consequently, the interior zone 2 with longitudinal dimension O(1) and transverse dimensions $O(\lambda)$ must be examined, in which in a first approximation the theory of elongated bodies is valid [5]. The boundary condition for the problem in zone 2 is obtained by merging the solutions in zones 1 and 2. In a first approximation the solution of the problem in zone 2 is independent of M_∞ and does not contain singularities in the variable x. This circumstance permits formulating the problem for the wake behind the wing: for $x = x_e$ (x_e is the coordinate of the wing trailing edge) the attached vortices on the wing are liberated and the two-dimensional problem of a wake in the presence of a fuselage and then without a fuselage is later solved.

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A study of the next approximation in zone 2 shows that it is singular in x at the point $x = x_e$. This circumstance is the foundation for introducing zone 3 with the characteristic dimensions $O(\lambda)$ in the neighborhood of the wing root. Analysis of the problem in zone 3 yields the result: the configuration of a vortex sheet and transverse velocity components are described correctly by the theory of elongated bodies in a first approximation while the longitudinal component of the velocity and the pressure cannot be obtained by using the theory of elongated bodies even in a first approximation. In order to find them it is necessary to solve a three-dimensional problem in zone 3 for a given configuration and vortex sheet intensity obtained by using the theory of elongated bodies. A three-dimensional linear problem for the Laplace equation is here obtained in the Goethert variables.

Therefore, the compressibility of the medium is manifest in problem 1 primarily at the wing root and it can be taken into account by using the Goethert rule even in the presence of flow separation.

Let us describe the algorithm of the solution of problem 1 which can be used to obtain a solution uniformly suitable in all the domains. The algorithm consists of two steps: in the first the two-dimensional nonlinear problem of the theory of elongated bodies is solved taking account of flow separation from the wing leading edges and interference between the leading edge and the fuselage, here the vortex sheet configuration and its intensity are found; in the second step a three-dimensional problem on the flow around the wing-fuselage combination is solved in the presence of a known incompressible fluid vortex sheet (in the Goethert variables). Asymptotic estimate of the errors for both the total and the distributed aerodynamic characteristics are given in [1].

Let us turn to a description of the results of an asymptotic analysis of problem 2 (the combination of a fuselage—finite-span wing—extension). The full exposition of the mathematical apparatus is presented in [2]. We introduce zone 1, the exterior flow domain with characteristic dimensions $O(1)$. Then for $\lambda_c \to 0$, $\varepsilon = O(\lambda_c)$ the fuselage and extension are mapped in the form of a segment of the x axis. In a first approximation a linear problem of separation-free flow around a wing without a fuselage and extension holds in zone 1; in the Goethert variables the perturbed potential satisfies the three-dimensional Laplace equation. Furthermore, as in problem 1 we consider the interior zone 2 with transverse dimensions $O(\lambda_c)$ and longitudinal dimension $O(1)$ in which the theory of elongated bodies is also valid in a first approximation. The boundary condition at infinity is obtained for the problem in zone 2 by using the merging of the solutions in zones 1 and 2. Therefore, the velocity at infinity in the plane problem is simply the wash at the point x obtained from the solution of the linear problem in zone 1.

As is shown in [2], the magnitude of the wash approaches infinity as the juncture domain of the extension and the wing is approached, consequently it is necessary to examine zone 3, the domain of the neighborhood of the juncture of extension and wing with characteristic dimension on the order of the transverse extension dimensions; its vortex sheet passes without a change of its configuration. Moreover, the influence of the vortex sheet on the flow velocity is small in it and linear theory is valid in a first approximation. This means that the Chaplygin—Zhukovskii condition is not satisfied on the side edges of the extension. This last circumstance results in the need to introduce narrow zones 4 in the neighborhood of the extension side edges, which are subzones of zone 3. The theory of elongated bodies is also valid there, where the velocity at infinity in this problem is obtained by using the merging of the solutions in zones 3 and 4 and is determined by the coefficient A for the singular term in the velocity in the linear problem in zone 3. Evidently A depends on M_∞. Therefore, the influence of compressibility on the vortex sheet configuration is felt in zones 2 and 4 in terms of the boundary conditions at infinity.

It is necessary to emphasize the special role of zone 4 in which a vortex sheet of elevated intensity is generated. Later a section of the vortex sheet forms a second nucleus wake above the wing. This effect was first analyzed theoretically in [6].

Thus by having information about the solution in zone 4 a problem about the wake above a wing (continuation of zone 2) can be formulated. Here the theory of elongated bodies is also valid and a two-dimensional problem on vortex sheet motion above a plane in the presence of a fuselage must be solved. Finally, there is still a characteristic domain of the wake behind the wing in the presence of a fuselage where the theory of elongated bodies is also valid.

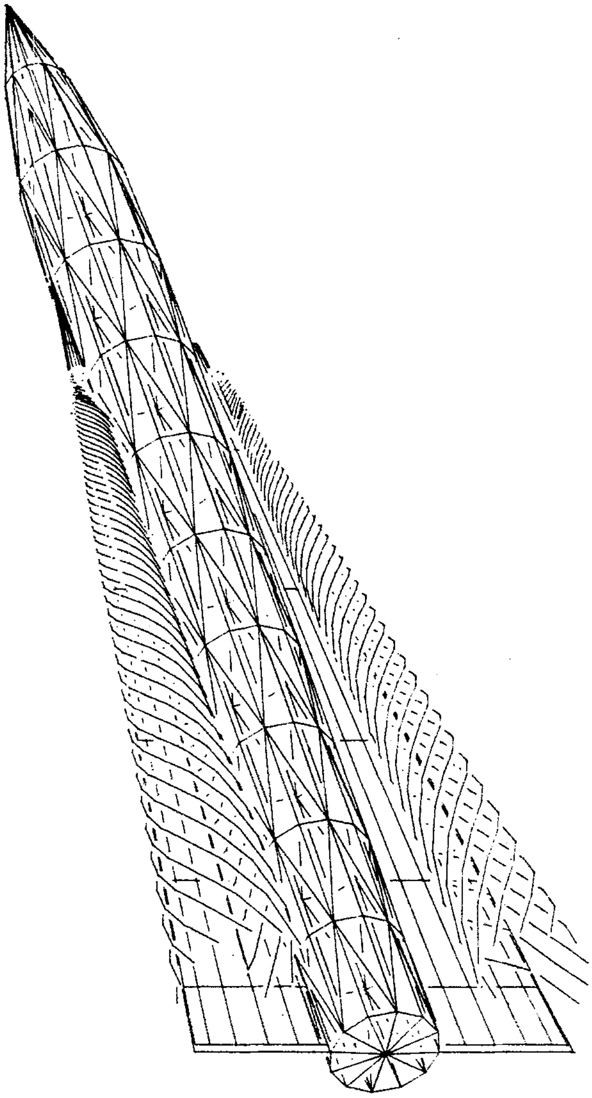

Fig. 1

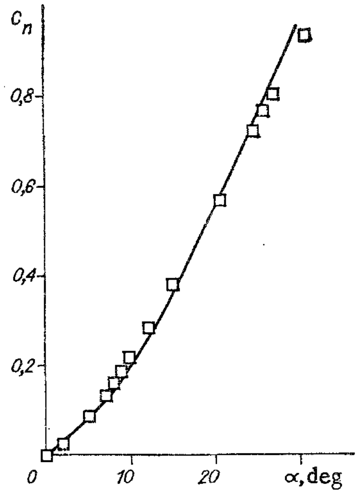

Fig. 2

Therefore, compressibility of the medium in problem 2 appears in zones 2 and 3 in terms of the boundary conditions at infinity. It hence follows that the vortex sheet configuration in problem 2 depends on M_∞ even in a first approximation.

On the basis of the asymptotic analysis, an algorithm can be formulated for the solution of problem 2 that would afford the possibility of obtaining a solution that is uniformly suitable in all domains. The algorithm consists of three steps. In the first, a linear problem in the Goethert variables is solved for the separation-free flow around a fuselage—wing—extension system and the A is determined for the singular term in the velocity in the neighborhood of the extension leading edges. Then the boundary condition for plane problems in zones 2 and 4 is found by means of the obtained value of A, as described in detail in [2]. Later a sequence of plane problems of the theory of elongated bodies on the development of sheets in the neighborhood of the extension and in the wake is solved. Finally, in the third step a three-dimensional problem for the Laplace equation in Goethert variables is solved for a known vortex sheet configuration and intensity and the aerodynamic characteristics, both the total and the distributed, are determined. Asymptotic estimates of the accuracy of the algorithm for the total and distributed aerodynamic characteristics are given in [2].

The method of vortex lattices [1-2] was used for the wing approximation in the numerical solution of the problem. The fuselage surface was represented by triangular panels on whose surface a source distribution with density constant over the panel was introduced.

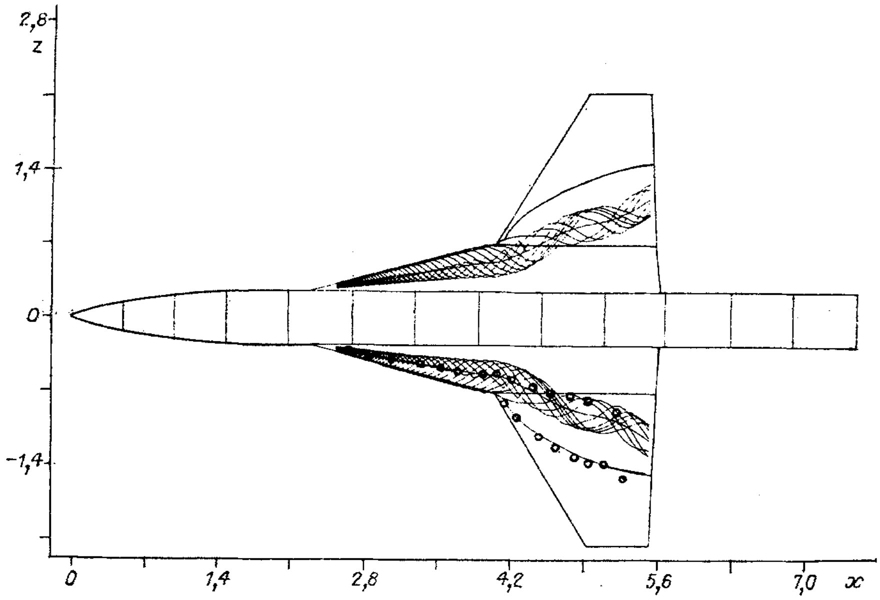

Fig. 3

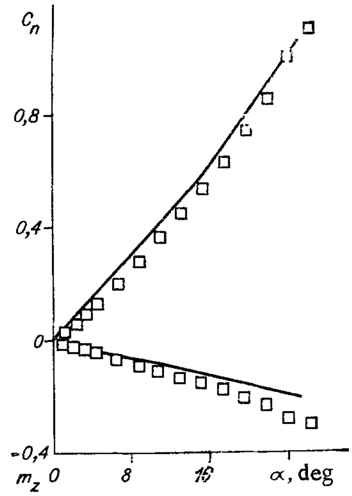

Fig. 4

3. Examples of the Computations

As an example of the application of the algorithm elucidated above we consider the results of computing the separation flow around the combination of a fuselage with a small-span triangular wing (Problem 1). The geometric parameters of the components under investigation are presented in [7]. The computation network (Fig. 1) contains 48 panels on the wing and 108 on the fuselage. The vortex sheet shedding from the wing leading edge was approximated by 12 vortex filaments. The shape of the vortex sheet above the wing for $M_\infty = 0.5$ and $\alpha = 15°$ is shown in Fig. 1. The computed dependence of the normal force coefficient on the angle of attack is compared with the data of experiment [7] of $M_\infty = 0.5$ in Fig. 2. The maximal divergence of the values of C_n obtained numerically and by experimental means does not exceed 5% in the range $0 < \alpha < 30°$.

Figure 3 shows the computed shape of the sheet above the wing with extension in the presence of a fuselage, and there are data of an experimental study (points) of this model of a flying vehicle in [8]. The computation is performed for $M_\infty = 0.5$ and $\alpha = 15°$. The mesh contains 112 panels on the wing and 132 on the fuselage. Two rarefaction peaks are noted on the wing surface in [8] and it is assumed that they correspond to the cores of the vortex sheet being shed from the leading edges of the extension and the leading edges of the main wing. However, as mentioned in Sec. 2, a vortex sheet of elevated intensity is generated in zone 4 of the juncture between the extension and the main wing, which then shapes a second core in the wake above the wing. Taking into account the circumstance that the leading edge sweepback of the wing under investigation is sufficiently small, it is assumed in the computation that there is no separation from the leading edge of the main wing. The wake above the wing is here modeled by a two-core vortex sheet.

Comparison of the locations of the rarefaction peaks and the computed wake configuration permits making another assumption about the origination of the second rarefaction peak: the second rarefaction peak noted is caused by passage of the second core of the sheet near the wing surface.

A comparison is made between the results of computing the flow characteristics around a fuselage—wing—extension combination and experimental data [9]. The same assumption about the flow diagram was made in the computations and a mesh with the same parameters was utilized, as in the preceding example. Figure 4 shows the dependences of the normal force coefficient C_n and the longitudinal moment m_z on α obtained by computational and experimental means for $M_\infty = 0.3$. It is seen that even in this case there is good agreement between the results of the computation and the data of the experiment.

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